Understanding Proportions by Designing Ramps

Standard: 6.NS.1.0/CCCS 7.RP Ratios and Proportional Relationships

Time: Two One Hour Class Periods

Materials: Pgs. 4 -7; graph paper; a ruler; calculators for practice page 8.

Warm up Key: I – 1) $\frac{11}{12}$ 2) $\frac{17}{21}$; II – B; III – 1) 2 to 5 2) 2:5 3) $\frac{2}{5}$; IV – Ex. 1) $\frac{6}{10}$ 2) $\frac{9}{15}$ 3) $\frac{12}{30}$

IV - Make the connection that proportions are equivalent fractions or ratios.

Introduction: **Grabber** - Tell the students that *Reliable Ramps* is looking for hard working junior engineers that follow directions well and are able to find the most efficient way to solve proportions for ramp design and construction.

Review – Think-Pair-Share (TPS) - Discuss what a ratio is and give examples. Ask how many have seen motorcycles jump from ramp to ramp. Have the students count the hands. Ask how many haven't seen it. Write the ratio three different ways (ex. Have to Haven't is 15 to 16, 15:16, 15/16). **TPS –** If this is typical, what would be the "Have to Haven't" ratio be in a class of 62? Show those ratios with an equal sign between them. Ex. $\frac{Have}{Haven't} = \frac{15}{16} = \frac{30}{32}$ This is a proportion. **TPS –** What is a proportion? **Explain that a proportion is a statement that two ratios are equivalent. (Note: Proportions will always graph through the origin)**

Lesson: **Analyzing a Graph and Extending Ratios** (Handout **"Reliable Ramps" Page 4**)

We Do – TPS - What would be the distance the motorcycle will travel if the ramp length is 5 meters? [By graph analysis it is 15 meters]. What would the ramp length need to be if you want to jump 120 meters? [40 meters].

You Try – In pairs students will fill-out the "Proportion Table" on Page 4.

TPS - How did you solve the proportions? [Interpreting the graph and finding the pattern of extended proportions in the table]. **TPS** - What challenges did the "Ramp Distance" of 70 meters present? [The graph isn't very helpful; therefore, the pattern had to be used to solve.] Today we will learn more methods that will be useful to solve certain proportions.

Have them record their answers on the **Page 6** "**Engineer Worksheet" 1-4**

Multiplying by an Equivalent Form of One (Handout **"Designing Ramps" Page 5**)

We Do

1) $\frac{Base}{Height} = \frac{4}{3} = \frac{b}{15}$

$\frac{5}{5} \bullet \frac{4}{3} = \frac{b}{15}$

$\frac{20}{15} = \frac{b}{15}$

$\therefore b = 20\ m$

2) $\frac{Base}{Height} = \frac{4}{3} = \frac{44}{h}$

$\frac{11}{11} \bullet \frac{4}{3} = \frac{44}{h}$

$\frac{44}{33} = \frac{44}{h}$

$\therefore h = 33\ m$

Always have students analyze the graph on page 5 before showing other methods. Explain that if two ratios are proportional, then the numerators and denominators will be equal. **TPS** – When would we use this method to solve proportions? [When it is easy to see the relationship.]

You Try – Page 5. Show calculations on **Page 6 "Engineer Worksheet" 5 – 8.**

Multiplying by the LCD, Least Common Denominator (Handout Page 5)

We Do

1) $\frac{4}{5}=\frac{26}{r}$

$\frac{5r}{1}\bullet\frac{4}{5}=\frac{26}{r}\bullet\frac{5r}{1}$

$\frac{20r}{5}=\frac{130r}{r}$

$4r=130$

$\frac{4r}{4}=\frac{130}{4}$

$r=32.5\,m$

2) $\frac{3}{5}=\frac{h}{7}$

$35\left(\frac{3}{5}\right)=35\left(\frac{h}{7}\right)$

$7\bullet 3=5\bullet h$

$21=5h$

$\frac{21}{5}=\frac{5h}{5}$

$4.2=h$

3) $\frac{3}{5}=\frac{21}{r}$

$5r\left(\frac{3}{5}\right)=5r\left(\frac{21}{r}\right)$

$3r=105$

$\frac{3r}{3}=\frac{105}{3}$

$r=35$

This is referred to as fraction busting. **TPS** – When would you use this method? [When I can't see the relationship right away.]

You Try – Page 5. Show calculations on **Page 7** "**Engineer Worksheet" 9 -12**

Cross Product Property (Handout Page 5)

We Do

1) $5r\left(\frac{4}{5}\right)=5r\left(\frac{36}{r}\right)$

$\frac{5\bullet r\bullet 4}{5}=\frac{5\bullet r\bullet 36}{r}$

$r\bullet 4=5\bullet 36$

$4r=180$

$\frac{4r}{4}=\frac{180}{4}$

$r=45$

2) $bd\left(\frac{a}{b}\right)=bd\left(\frac{c}{d}\right)$

$\frac{abd}{b}=\frac{bdc}{d}$

$ad=bc$

$b\ and\ d\neq 0$

3) $\frac{3}{8}=\frac{h}{20}$

$20\bullet 8\left(\frac{3}{8}\right)=\left(\frac{h}{20}\right)20\bullet 8$

$20\bullet 3=8\bullet h$

$60=8h$

$h=7.5$

4) $\frac{h}{20}=\frac{3}{5}$

$5h=60$

$\frac{5h}{5}=\frac{60}{5}$

$h=12\,m$

5) $\frac{a}{b}=\frac{c}{d}$

$ad=bc$

$b\ and\ d\neq 0$

TPS – When would you use this method? [When the relations isn't easy to find.]
The Cross Product Property works because each side is being multiplied by a common denominator found by multiplying the two denominators together (see #1, #2, and #3). You may only want to show this method the first year proportions are introduced.

You Try – Page 5. Show calculations on **Page 7** "**Engineer Worksheet" 13 - 16.**

Warm-Up

CST/CAHSEE: 4.NS.1.6/CCCS 5.NF.3

Which of the following is equivalent to $\frac{5}{2}$?

A 2.25

B 2.5

C 5.2

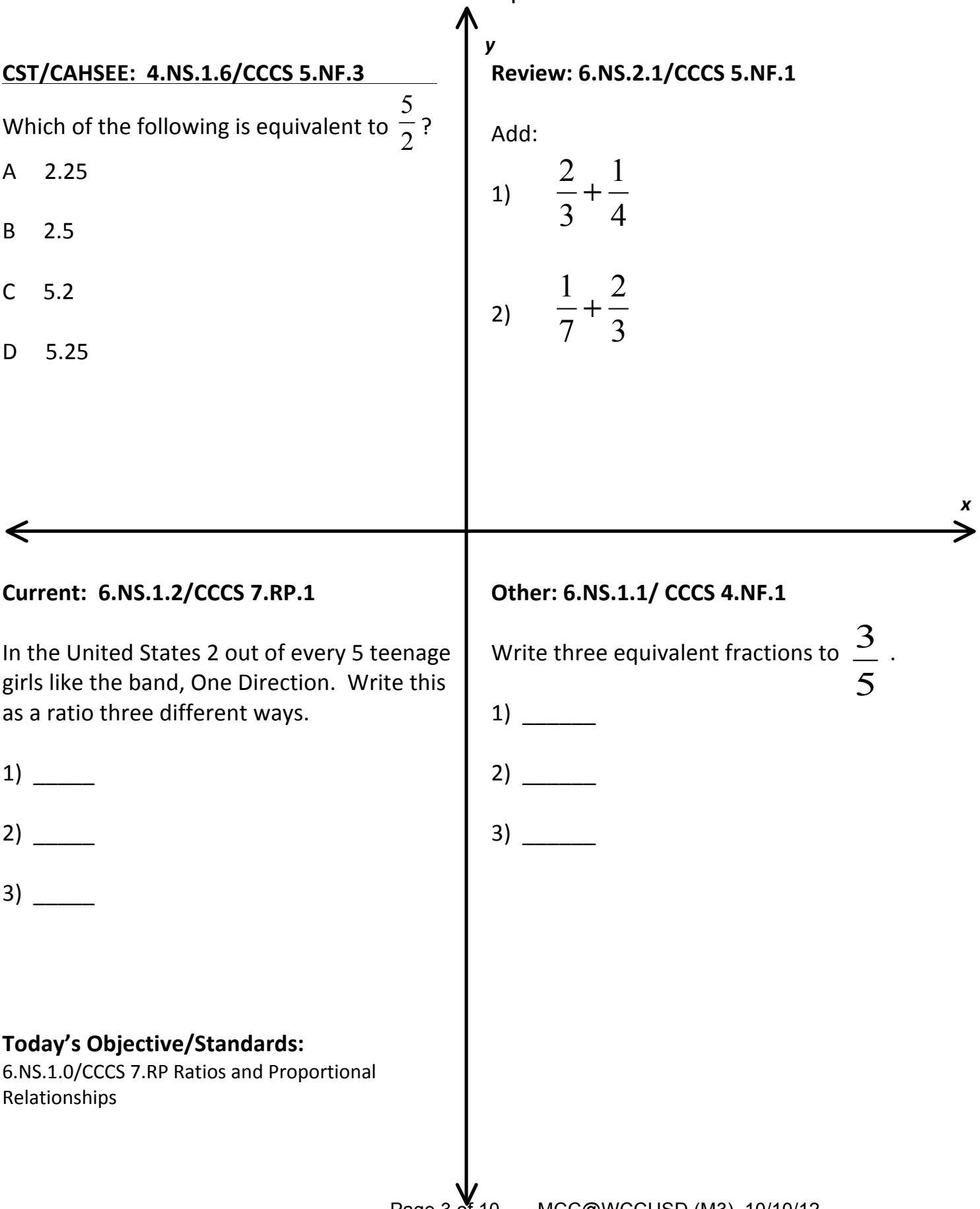
D 5.25

Review: 6.NS.2.1/CCCS 5.NF.1

Add:

1) $\frac{2}{3}+\frac{1}{4}$

2) $\frac{1}{7}+\frac{2}{3}$



Current: 6.NS.1.2/CCCS 7.RP.1

In the United States 2 out of every 5 teenage girls like the band, One Direction. Write this as a ratio three different ways.

1) _____

2) _____

3) _____

Other: 6.NS.1.1/ CCCS 4.NF.1

Write three equivalent fractions to $\frac{3}{5}$.

1) _____

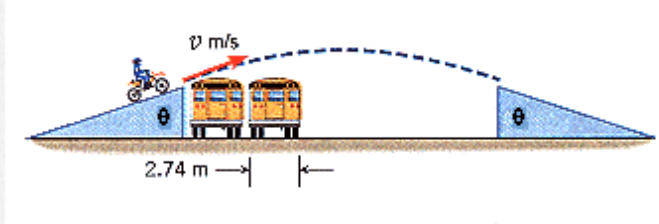
2) _____

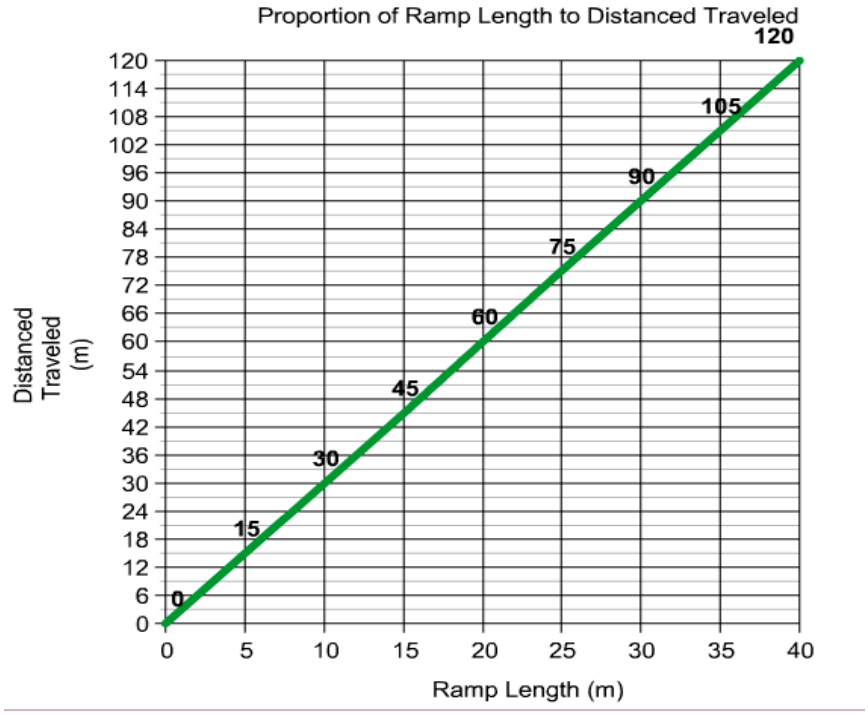
3) _____

Today's Objective/Standards:
6.NS.1.0/CCCS 7.RP Ratios and Proportional Relationships

Name_____ Per. _____

Reliable Ramps



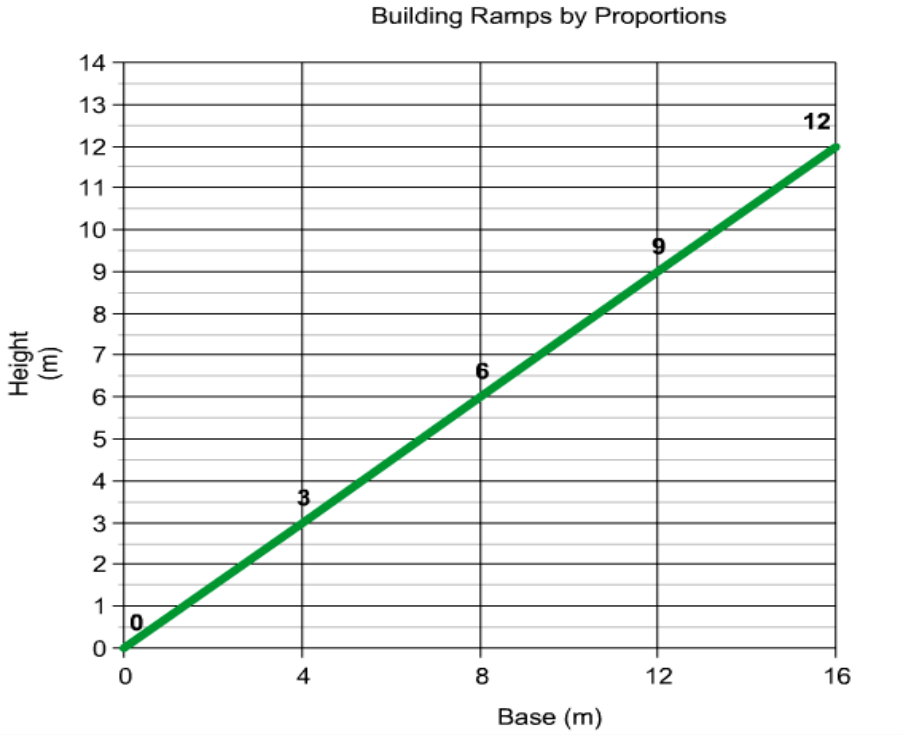


Proportion Table

Ramp Length to **Distance Traveled** is **1/3** **Worksheet 1 - 4**

Ramp Length		10 meters	15 meters			70 meters
Distance Traveled	15 meters			60 meters	90 meters	

Designing Ramps by Proportions



Base to **Height** Ratio is **4**/**3** **Worksheet 5 - 8**

Base	8 meters			40 meters
Height		12 meters	27 meters	

Base to **Ramp Length** Ratio is **4**/**5** **Worksheet 9 - 12**

Base	1 meter	16 meters		
Ramp Length			30 meters	60 meters

Height to **Ramp Length** Ratio is **3**/**5** **Worksheet 13 - 16**

Height	12 meters	33 meters		
Ramp Length			67 meters	92 meters

Engineer Worksheet **Name_____ Per._____**

Solve by **Analyzing a Graph and Extending Ratios**

1) $\frac{Ramp}{Dist.} = \frac{1}{3} = \frac{r}{15}$ $r =$ _____

2) $\frac{Ramp}{Dist.} = \frac{1}{3} = \frac{10}{d}$ $d =$ _____

3) $\frac{Ramp}{Dist.} = \frac{1}{3} = \frac{15}{d}$ $d =$ _____

4) $\frac{Ramp}{Dist.} = \frac{r}{60} = \frac{1}{3}$ $r =$ _____

Solve by **Multiplying by An Equivalent Form of One** (show all of your calculations)

5) $\frac{Base}{Height} = \frac{4}{3} = \frac{8}{h}$ $h =$ _____

6) $\frac{Base}{Height} = \frac{4}{3} = \frac{b}{12}$ $b =$ _____

7) $\frac{Base}{Height} = \frac{4}{3} = \frac{b}{27}$ $b =$ _____

8) $\frac{Base}{Height} = \frac{40}{h} = \frac{4}{3}$ $h =$ _____

Solve by **Multiplying by the LCD** (show all of your calculations)

9) $\frac{Base}{Ramp} = \frac{4}{5} = \frac{1}{r}$ $r =$ _____

10) $\frac{Base}{Ramp} = \frac{4}{5} = \frac{16}{r}$ $r =$ _____

11) $\frac{Base}{Ramp} = \frac{4}{5} = \frac{b}{30}$ $b =$ _____

12) $\frac{Base}{Ramp} = \frac{4}{5} = \frac{b}{60}$ $b =$ _____

Solve by any method (show all of your calculations)

13) $\frac{Height}{Ramp} = \frac{3}{5} = \frac{12}{r}$ $r =$ _____

14) $\frac{Height}{Ramp} = \frac{3}{5} = \frac{33}{r}$ $r =$ _____

15) $\frac{Height}{Ramp} = \frac{3}{5} = \frac{h}{67}$ $h =$ _____

16) $\frac{Height}{Ramp} = \frac{h}{92} = \frac{3}{5}$ $h =$ _____

Name_____ Date_____ Per._____

Practice (choose any method to solve the proportions)

	3/1	5/4	4/3
Distance Needing to Travel	**Length of Ramp**	**Base**	**Height**
18 meters (6 Buses)	$\frac{d}{r}=\frac{3}{1}=\frac{18}{r}$ r = ____	$\frac{r}{b}=\frac{5}{4}=\frac{6}{b}$ b = ____	$\frac{b}{h}=\frac{4}{3}=\frac{4.8}{h}$ h = ____
30 meters (10 Buses)			
48 meters (16 Buses)			
60 meters (20 Buses)			
81 meters (27 Buses)			
93 meters (31 Buses)			
165 meters (55 Buses)			

Extension:

You're the Engineer! Graph a side view of a motorcycle ramp with a length of 20 meters. Show the supports that will be built vertically every 4 meters from the base to the ramp. Label your base, length, and ramp length. Use the scale 1 cm = 1 m. Build a model of the ramp using cardboard, wood, or any other materials you want.

Reliable Ramps – Page 6 Solve by **Analyzing a Graph and Extending Ratios**

1) $\frac{1}{3}=\frac{r}{15}$ $r=5\,m$

2) $\frac{1}{3}=\frac{10}{d}$ $d=30\,m$

3) $\frac{1}{3}=\frac{15}{d}$ $d=45\,m$

4) $\frac{r}{60}=\frac{1}{3}$ $r=20\,m$

Ramp Length	5 meters	10 meters	15 meters	20 meters	30 meters	70 meters
Distance Traveled	15 meters	30 meters	45 meters	60 meters	90 meters	210 meters

Designing Ramps by Proportions Page 7 Solve by **Multiplying by an Equivalent Form of One**

5) $\frac{4}{3}=\frac{8}{h}$

$\frac{2}{2}\bullet\frac{4}{3}=\frac{8}{6}$

$\therefore h=6m$

6) $\frac{4}{3}=\frac{b}{12}$

$\frac{4}{4}\bullet\frac{4}{3}=\frac{b}{12}$

$\therefore b=16m$

7) $\frac{4}{3}=\frac{b}{27}$

$\frac{9}{9}\bullet\frac{4}{3}=\frac{b}{27}$

$\therefore h=36m$

8) $\frac{40}{h}=\frac{4}{3}$

$\frac{40}{30}=\frac{4}{3}\bullet\frac{10}{10}$

$\therefore h=30m$

Solve by **Multiplying by LCD**

9) $\frac{4}{5}=\frac{1}{r}$

$\frac{5r}{1}\bullet\frac{4}{5}=\frac{1}{r}\bullet\frac{5r}{1}$

$\frac{20r}{5}=\frac{5r}{r}$

$4r=5$

$\frac{4r}{4}=\frac{5}{4}$

$r=1.25m$

10) $\frac{4}{5}=\frac{16}{r}$

$\frac{5r}{1}\bullet\frac{4}{5}=\frac{16}{r}\bullet\frac{5r}{1}$

$\frac{20r}{5}=\frac{80r}{r}$

$4r=80$

$\frac{4r}{4}=\frac{80}{4}$

$r=20\,m$

11) $\frac{4}{5}=\frac{b}{30}$

$\frac{30}{1}\bullet\frac{4}{5}=\frac{b}{30}\bullet\frac{30}{1}$

$\frac{120}{5}=\frac{30b}{30}$

$24=b$

$b=24\,m$

12) $\frac{4}{5}=\frac{b}{60}$

$\frac{60}{1}\bullet\frac{4}{5}=\frac{b}{60}\bullet\frac{60}{1}$

$\frac{240}{5}=\frac{60b}{60}$

$48=b$

$b=48\,m$

Solve Using a Variety of Methods

13) $\frac{3}{5}=\frac{12}{r}$

$5r\bullet\left(\frac{3}{5}\right)=5r\bullet\left(\frac{12}{r}\right)$

$3r=60$

$\frac{3r}{3}=\frac{60}{3}$

$r=20\,m$

14) $\frac{3}{5}=\frac{33}{r}$

$\frac{r}{r}\bullet\frac{3}{5}=\frac{33}{r}\bullet\frac{5}{5}$

$\frac{3r}{5r}=\frac{165}{5r}$

$\therefore 3r=165$

$\frac{3r}{3}=\frac{165}{3}$

$r=55\,m$

15) $\frac{3}{5}=\frac{h}{67}$

$67\bullet 3=5\bullet h$

$201=5h$

$\frac{201}{5}=\frac{5h}{5}$

$40.2=h$

$h=40.2\,m$

16) $\frac{h}{92}=\frac{3}{5}$

$5\bullet h=92\bullet 3$

$5h=276$

$\frac{5h}{5}=\frac{276}{5}$

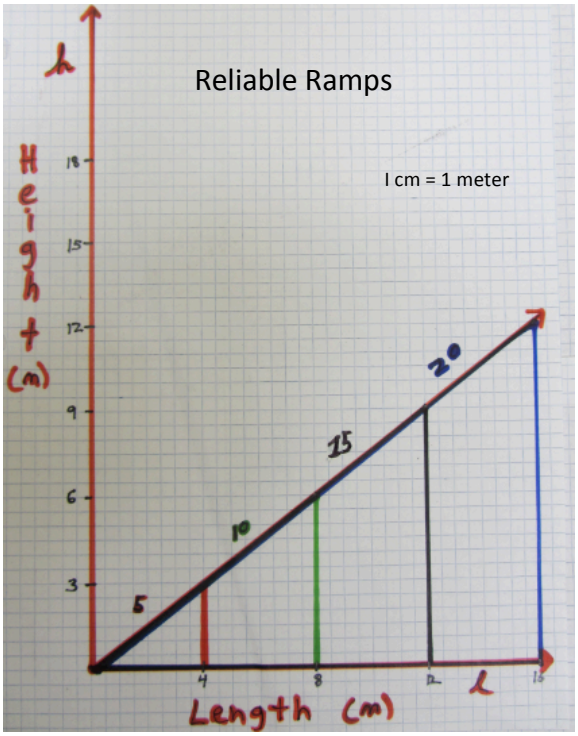
$h=55.2\,m$

When debriefing the practice, have students share with each other how they solved their problems. Select a few problems and have the students show, side by side, how they solved the proportions using different methods.

Key to Practice

Distance Needing to Travel	Length of Ramp	Base	Height
18 meters (6 Buses)	r = 6 m	b = 4.8 m	h = 3.6 m
30 meters (10 Buses)	r = 10 m	b = 8 m	h = 6 m
48 meters (16 Buses)	r = 16 m	b = 12.8 m	h = 9.6 m
60 meters (20 Buses)	r = 20 m	b = 16 m	h = 12 m
81 meters (27 Buses)	r = 27 m	b = 21.6 m	h = 16.2 m
93 meters (31 Buses)	r = 31 m	b = 24.8 m	h = 18.6 m
165 meters (55 Buses)	r = 55 m	b = 44 m	h = 33 m

Key to Extension



Other Applications

This lesson can be a springboard for many other lessons:

1) Slope – rise to run ratios

2) Similar Triangles

3) Pythagorean Theorem $a^2 + b^2 = c^2$

4) Measurement

5) Graphing

6) Scale Drawing